The 23rd Nordic Mathematical Contest

Thursday April 2, 2009

English version

Time allowed is 4 hours. Each problem is worth 5 points. The only permitted aids are writing and drawing tools.

Problem 1

A point P is chosen in an arbitrary triangle. Three lines are drawn through P which are parallel to the sides of the triangle. The lines divide the triangle into three smaller triangles and three parallelograms. Let f be the ratio between the total area of the three smaller triangles and the area of the given triangle. Show that $f \geq \frac{1}{3}$ and determine those points P for which $f = \frac{1}{3}$.

Problem 2

On a faded piece of paper it is possible, with some effort, to discern the following:

$$(x^2 + x + a)(x^{15} - \ldots) \; = \; x^{17} + x^{13} + x^5 - 90x^4 + x - 90.$$

Some parts have got lost, partly the constant term of the first factor of the left side, partly the main part of the other factor. It would be possible to restore the polynomial forming the other factor, but we restrict ourselves to asking the question: What is the value of the constant term a? We assume that all polynomials in the statement above have only integer coefficients.

Problem 3

The integers 1, 2, 3, 4 and 5 are written on a blackboard. It is allowed to wipe out two integers a and b and replace them with $a + b$ and ab. Is it possible, by repeating this procedure, to reach a situation where three of the five integers on the blackboard are 2009?

Problem 4

There are 32 competitors in a tournament. No two of them are equal in playing strength, and in a one against one match the better one always wins. Show that the gold, silver, and bronze medal winners can be found in 39 matches.